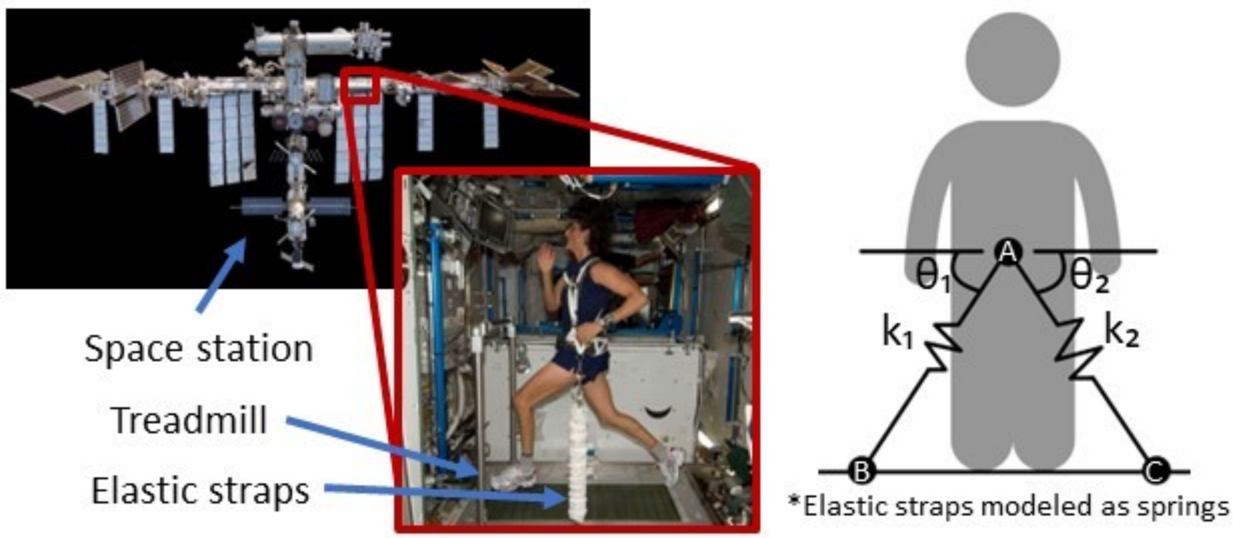
Name: _______________
Group members: ____________________ ____________________ ____________________

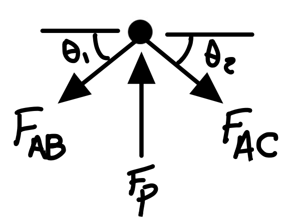
TAM 210/211 - Worksheet 3

Objectives:

- Use free body diagrams and equilibrium equations to determine forces in cables and springs.
- Explore an experimental setup to verify theoretical findings.



1) a) On the point below, draw a free body diagram for point A. Denote the force in spring AC as F_{AC}, the force in spring AB as F_{AB}, and the reaction force as F_P.



b) Use the equilibrium equations $\sum \mathbf{F} = \mathbf{0}$ to determine F_{AC} and F_{AB}. Your answers should be functions of F_P, θ_1 and θ_2.

$$\Sigma F_x = 0 : F_{AC}\cos\theta_2 - F_{AB}\cos\theta_1 = 0 \quad \text{—— } ①$$

$$\Sigma F_y = 0 : -F_{AB}\sin\theta_1 - F_{AC}\sin\theta_2 + F_p = 0 \quad \text{—— } ②$$

from ① $F_{AB} = F_{AC}\dfrac{\cos\theta_2}{\cos\theta_1}$ Using this in ② we get

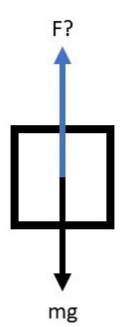
$$F_{AC} = \frac{F_p\cos\theta_1}{\sin\theta_1\cos\theta_2 + \sin\theta_2\cos\theta_1} \qquad F_{AB} = \frac{F_p\cos\theta_2}{\sin\theta_1\cos\theta_2 + \sin\theta_2\cos\theta_1}$$

2) Fun fact: In 1983, Sally Ride was the first American woman to fly in space at the age of 32. During her time as an astronaut, she weighed 511 N. How much force would the springs need so she could stand on the treadmill with her full body weight stimulated? $\theta_1 = 70°$ and $\theta_2 = 70°$.

$F_{AC} = 271.9\,N$ $F_{AB} = 271.9\,N$

3) Use an experimental setup to validate your findings above. Start by calibrating the springs. Make sure to have the reading mark set to "zero" when the spring is hanging unloaded at a vertical position.

a) Find the force required to simulate zero gravity by using a simple spring.

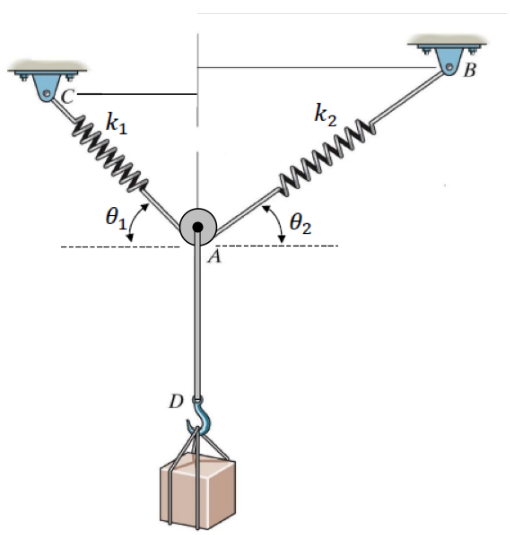


$F = mg$

b) Use the peg-board, spring, ring, bolts and the weight object to reproduce Question 1 setup. Use the angle measurements and the mass of your object to predict your spring reading using your equation from Question 1(b). Check your solution with actual spring readings.

Back on Earth, springs are used in various other applications. The following problems are examples of these applications.

4) In this next experimental setup, the springs are fixed at uneven positions (different heights) again, but the springs are no longer connected to each other via a ring. Instead, connect the springs using a piece of string, to model the cable that goes through the pulley at A. Use a bobbin to represent the frictionless pulley.



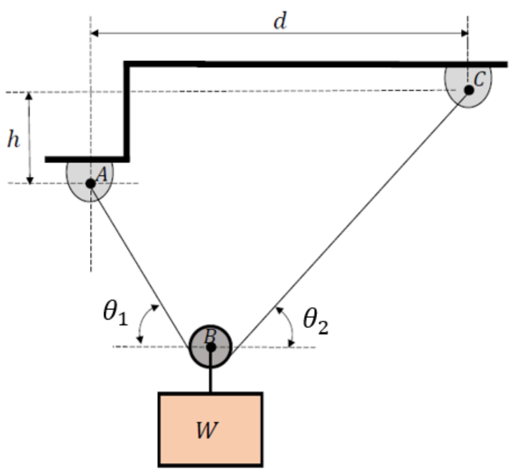
a) After hanging your object, what do you notice about angles θ_1 and θ_2? What does this tell you about the forces acting on the two ends of the string?

$\theta_1 = \theta_2 \Rightarrow \sum F_n = 0 \quad F_{AB} \cos\theta_2 = F_{AC} \cos\theta_1$

$\Rightarrow F_{AB} = F_{AC}$

b) Use the angle measurements and the mass of your object to predict your spring reading using your equations from Question 1(b). Check your solution with actual spring readings.

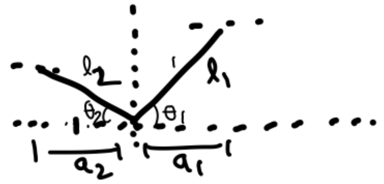
5) Another setup is illustrated below with string ABC of length L.



a) What happens to angles θ_1 and θ_2 when the weight W is changed by changing the object? Why?

θ_1 & θ_2 donot change with weight. They remain constant and equal.

$\theta_1 = \theta_2$



$l = l_1 + l_2 \qquad d = a_1 + a_2$

b) How do angles θ_1 and θ_2 relate to each other? Express the angles **in terms of the given symbolic variables and dimensions** (neglect the size of the frictionless pulley B). How does your theoretical expression validate your conclusions in part (a)?

Tension in a string remains same. $F_{AB} = F_{AC}$

$\Rightarrow \theta_2 = \theta_1 = \theta. \qquad l_1 = \dfrac{a_1}{\cos\theta_1} \qquad l_2 = \dfrac{a_2}{\cos\theta_2} \Rightarrow l_1 + l_2 = \dfrac{a_1 + a_2}{\cos\theta}$

$\Rightarrow l = \dfrac{d}{\cos\theta} \Rightarrow \theta = \cos^{-1}\left(\dfrac{d}{l}\right).$

c) Express the forces along AB and BC as Cartesian vectors in terms of the given symbolic variables and dimensions.

$F_{AB} = F_{BC} = \dfrac{W}{2\sin\theta}$

$F_{AB} = -F_{AB}\cos\theta_1\,\hat{i} + F_{AB}\sin\theta_1\,\hat{j} = \dfrac{-Wd}{2\sqrt{l^2 - d^2}}\,\hat{i} + \dfrac{W}{2}\,\hat{j}$

$F_{BC} = F_{BC}\cos\theta_2\,\hat{i} + F_{BC}\sin\theta_2\,\hat{j} = \dfrac{Wd}{2\sqrt{l^2 - d^2}}\,\hat{i} + \dfrac{W}{2}\,\hat{j}$

d) If the string ABC were shorter, how would angles θ_1 and θ_2 and the forces along AB and BC change?

$\theta = \cos^{-1}\left(\dfrac{d}{l}\right)$. If l is reduced $\theta_1 = \theta_2 = \theta$ will reduce

$F_{AB} = F_{BC} = \dfrac{W}{2\sin\theta} \Rightarrow$ as θ is reduced $\sin\theta$ reduces

and as a result F_{AB} & F_{BC} will increase.

e) What implications does part (d) have on the design of systems with different string lengths in terms of the required strengths of the strings?

strings with shorter length should have more strength, as the tension developed in them will be larger.